

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Number Sense and Computation	1	Understand numbers, ways of representing numbers, relationships among numbers, and number systems within the Real Number System
Mathematics	9-12	Number Sense and Computation	2	Simplify square roots using factors
Mathematics	9-12	Number Sense and Computation	3	Understand and use the distributive, associative, and commutative properties
Mathematics	9-12	Number Sense and Computation	4	Use the laws of exponents for rational exponents
Mathematics	9-12	Number Sense and Computation	5	Explain how the definition of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents
Mathematics	9-12	Number Sense and Computation	6	Rewrite expressions involving radicals and rational exponents using the properties of exponents
Mathematics	9-12	Number Sense and Computation	7	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Algebra and Functions	1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
Mathematics	9-12	Algebra and Functions	2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$
Mathematics	9-12	Algebra and Functions	3	Find powers and roots of monomials (only when the answer has an integer exponent)
Mathematics	9-12	Algebra and Functions	4	Divide polynomials by monomials
Mathematics	9-12	Algebra and Functions	5	Find a common monomial factor in a polynomial
Mathematics	9-12	Algebra and Functions	6	Add, subtract and multiply polynomials and divide polynomials by monomials.
Mathematics	9-12	Algebra and Functions	7	Factor common terms from polynomials and factor quadratic expressions
Mathematics	9-12	Algebra and Functions	8	Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree
Mathematics	9-12	Algebra and Functions	9	Factor polynomials completely and solve polynomial equations by factoring
Mathematics	9-12	Algebra and Functions	10	Use graphing technology to find approximate solutions for polynomial equations
Mathematics	9-12	Algebra and Functions	11	Solve problems that can be represented or modeled using polynomial equations, interpret the solutions and determine whether the solutions are reasonable
Mathematics	9-12	Algebra and Functions	12	Find a polynomial function of lowest degree with real coefficients when given its roots. Solve problems by using the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression
Mathematics	9-12	Algebra and Functions	13	Interpret parts of an expression, such as terms, factors, and coefficients that represent a quantity in terms of its context, use the structure of an expression to identify ways to rewrite it
Mathematics	9-12	Algebra and Functions	14	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression
Mathematics	9-12	Algebra and Functions	15	Factor a quadratic expression to reveal the zeros of the function it defines
Mathematics	9-12	Algebra and Functions	16	Use the properties of exponents to transform expressions for exponential functions
Mathematics	9-12	Algebra and Functions	17	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system
Mathematics	9-12	Algebra and Functions	18	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions
Mathematics	9-12	Algebra and Functions	19	Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions with linear and quadratic denominators, including denominators with negative exponents
Mathematics	9-12	Algebra and Functions	20	Simplify radical expressions involving square roots
Mathematics	9-12	Algebra and Functions	21	Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph and the factors of a polynomial expression
Mathematics	9-12	Algebra and Functions	22	Understand and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression
Mathematics	9-12	Algebra and Functions	23	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions
Mathematics	9-12	Algebra and Functions	24	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales
Mathematics	9-12	Algebra and Functions	25	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context
Mathematics	9-12	Algebra and Functions	26	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Algebra and Functions	27	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method
Mathematics	9-12	Algebra and Functions	28	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise
Mathematics	9-12	Algebra and Functions	29	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters
Mathematics	9-12	Algebra and Functions	30	Solve quadratic equations in one variable
Mathematics	9-12	Algebra and Functions	31	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form
Mathematics	9-12	Algebra and Functions	32	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions
Mathematics	9-12	Algebra and Functions	33	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables
Mathematics	9-12	Algebra and Functions	34	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$
Mathematics	9-12	Algebra and Functions	35	Represent a system of linear equations as a single matrix equation in a vector variable
Mathematics	9-12	Algebra and Functions	36	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line)
Mathematics	9-12	Algebra and Functions	37	Write the equation of a line that models a data set and use the equation (or the graph of the equation) to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change
Mathematics	9-12	Algebra and Functions	38	Use graphing technology to find approximate solutions of quadratic and cubic equations
Mathematics	9-12	Algebra and Functions	39	Identify the domain and range of relations represented by tables, graphs, words and equations
Mathematics	9-12	Algebra and Functions	40	Graph linear equations and show they have constant rates of change
Mathematics	9-12	Algebra and Functions	41	Determine the slope, x-intercept and y-intercept of a line given its graph, its equation or two points on the line. Then determine the equation of a line given sufficient information
Mathematics	9-12	Algebra and Functions	42	Write, interpret and translate among equivalent forms of equations for linear functions (i.e., slope-intercept, point-slope and standard). Recognize that equivalent forms reveal more or less information about a given situation
Mathematics	9-12	Algebra and Functions	43	Solve problems that can be modeled using linear equations and inequalities, interpret the solutions and determine whether the solutions are reasonable
Mathematics	9-12	Algebra and Functions	44	Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination
Mathematics	9-12	Algebra and Functions	45	Solve problems that can be modeled using quadratic equations, interpret the solutions and determine whether the solutions are reasonable
Mathematics	9-12	Algebra and Functions	46	Sketch and interpret linear and non-linear graphs representing given situations and identify independent and dependent variables
Mathematics	9-12	Algebra and Functions	47	Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations
Mathematics	9-12	Algebra and Functions	48	Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution as the original. Understand that similar logic applies to solving systems of equations simultaneously
Mathematics	9-12	Algebra and Functions	49	Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable
Mathematics	9-12	Algebra and Functions	50	Solve equations and inequalities involving the absolute value of a linear function
Mathematics	9-12	Algebra and Functions	51	Solve equations that contain radical expressions and identify extraneous roots when they occur
Mathematics	9-12	Algebra and Functions	52	Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions and determine whether the solutions are reasonable

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Algebra and Functions	53	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes
Mathematics	9-12	Algebra and Functions	54	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph
Mathematics	9-12	Algebra and Functions	55	Graph quadratic, cubic, square root, cube root, and piecewise-defined functions, including step functions and absolute value functions
Mathematics	9-12	Algebra and Functions	56	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function
Mathematics	9-12	Algebra and Functions	57	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context
Mathematics	9-12	Algebra and Functions	58	Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay
Mathematics	9-12	Algebra and Functions	59	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)
Mathematics	9-12	Algebra and Functions	60	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse
Mathematics	9-12	Algebra and Functions	61	Find and verify by composition that one function is the inverse of another
Mathematics	9-12	Algebra and Functions	62	Read values of an inverse function from a graph or a table, given that the function has an inverse
Mathematics	9-12	Algebra and Functions	63	Produce an invertible function from a non-invertible function by restricting the domain
Mathematics	9-12	Algebra and Functions	64	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle
Mathematics	9-12	Algebra and Functions	65	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions
Mathematics	9-12	Algebra and Functions	66	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline
Mathematics	9-12	Algebra and Functions	67	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed
Mathematics	9-12	Algebra and Functions	68	Determine whether a relation represented by a table, graph, words or equation is a function or not a function. Translate among tables, graphs, words and equations
Mathematics	9-12	Algebra and Functions	69	Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs
Mathematics	9-12	Algebra and Functions	70	Recognize and graph various types of functions, including polynomial, rational, and algebraic functions
Mathematics	9-12	Algebra and Functions	71	Use function notation. Add, subtract, multiply, and divide pairs of functions
Mathematics	9-12	Algebra and Functions	72	Understand composition of functions and combine functions by composition
Mathematics	9-12	Algebra and Functions	73	Graph relations and functions with and without graphing technology
Mathematics	9-12	Algebra and Functions	74	Solve and Graph equations and inequalities involving the absolute value of a linear function
Mathematics	9-12	Algebra and Functions	75	Analyze, describe and sketch graphs of quadratic functions and include the lines of symmetry
Mathematics	9-12	Algebra and Functions	76	Analyze, describe and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior
Mathematics	9-12	Algebra and Functions	77	Distinguish between situations that can be modeled with linear functions and with exponential functions
Mathematics	9-12	Algebra and Functions	78	Understand that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals
Mathematics	9-12	Algebra and Functions	79	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function
Mathematics	9-12	Algebra and Functions	80	For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology
Mathematics	9-12	Algebra and Functions	81	Interpret the parameters in a linear or exponential function in terms of a context
Mathematics	9-12	Algebra and Functions	82	Use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents
Mathematics	9-12	Algebra and Functions	83	Understand and use negative and fractional exponents

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Algebra and Functions	84	Prove simple laws of logarithms
Mathematics	9-12	Algebra and Functions	85	Solve logarithmic and exponential equations and inequalities
Mathematics	9-12	Algebra and Functions	86	Use the definition of logarithms to convert logarithms from one base to another
Mathematics	9-12	Algebra and Functions	87	Use the properties of logarithms to simplify logarithmic expressions and to find their approximate values
Mathematics	9-12	Algebra and Functions	88	Understand the properties of rational exponents and use the properties to simplify, multiply, divide and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions
Mathematics	9-12	Algebra and Functions	89	Analyze, describe and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior
Mathematics	9-12	Algebra and Functions	90	Analyze, describe and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior
Mathematics	9-12	Algebra and Functions	91	Know that the inverse of an exponential function is a logarithm. Use laws of exponents to derive laws of logarithms. Use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems
Mathematics	9-12	Algebra and Functions	92	Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable. Use technology as appropriate
Mathematics	9-12	Algebra and Functions	93	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real
Mathematics	9-12	Algebra and Functions	94	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers
Mathematics	9-12	Algebra and Functions	95	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers
Mathematics	9-12	Algebra and Functions	96	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number
Mathematics	9-12	Algebra and Functions	97	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation
Mathematics	9-12	Algebra and Functions	98	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints
Mathematics	9-12	Algebra and Functions	99	Solve quadratic equations with real coefficients that have complex solutions
Mathematics	9-12	Algebra and Functions	100	Extend polynomial identities to the complex numbers
Mathematics	9-12	Algebra and Functions	101	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials
Mathematics	9-12	Algebra and Functions	102	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle
Mathematics	9-12	Algebra and Functions	103	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.
Mathematics	9-12	Algebra and Functions	104	Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b
Mathematics	9-12	Algebra and Functions	105	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater)
Mathematics	9-12	Algebra and Functions	106	Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions
Mathematics	9-12	Algebra and Functions	107	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes
Mathematics	9-12	Algebra and Functions	108	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context
Mathematics	9-12	Algebra and Functions	109	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Algebra and Functions	110	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
Mathematics	9-12	Algebra and Functions	111	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them
Mathematics	9-12	Algebra and Functions	112	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents
Mathematics	9-12	Algebra and Functions	113	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another
Mathematics	9-12	Algebra and Functions	114	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle
Mathematics	9-12	Algebra and Functions	115	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x , $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number
Mathematics	9-12	Algebra and Functions	116	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context
Mathematics	9-12	Algebra and Functions	117	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle
Mathematics	9-12	Algebra and Functions	118	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems
Mathematics	9-12	Algebra and Functions	119	Find solution sets of linear inequalities when possible numbers are given for the variable
Mathematics	9-12	Algebra and Functions	120	Solve combined linear inequalities using properties of order
Mathematics	9-12	Algebra and Functions	121	Sketch and Interpret a reasonable graph for a given relationship
Mathematics	9-12	Algebra and Functions	122	Graph a linear inequality in two variables
Mathematics	9-12	Algebra and Functions	123	Factor the difference of two squares and other quadratics
Mathematics	9-12	Algebra and Functions	124	Simplify and solve algebraic ratios and proportions
Mathematics	9-12	Algebra and Functions	125	Graph with and without technology the solution set for a pair of linear inequalities in two variables. Use the graph to find the solution set
Mathematics	9-12	Algebra and Functions	126	Solve problems that can be modeled using pairs of linear equations in two variables, interpret the solutions and determine whether the solutions are reasonable
Mathematics	9-12	Algebra and Functions	127	Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square and by using the quadratic formula
Mathematics	9-12	Algebra and Functions	128	Write and interpret statements of the form “if – then” and “if and only if.”
Mathematics	9-12	Algebra and Functions	129	Solve an inequality by examining the graph
Mathematics	9-12	Algebra and Functions	130	Use substitution, elimination, and matrices to solve systems of two or three linear equations in two or three variables using Appropriate Technology
Mathematics	9-12	Algebra and Functions	131	Write the equations of conic sections (circle, ellipse, parabola, and hyperbola)
Mathematics	9-12	Algebra and Functions	132	Graph conic sections
Mathematics	9-12	Algebra and Functions	133	Add, subtract, multiply, divide, and simplify algebraic fractions
Mathematics	9-12	Algebra and Functions	134	Solve equations involving algebraic fractions
Mathematics	9-12	Algebra and Functions	135	Solve word problems involving fractional equations
Mathematics	9-12	Algebra and Functions	136	Solve problems of direct, inverse, and joint variation
Mathematics	9-12	Algebra and Functions	137	Define, add, subtract, multiply and divide complex numbers. Represent complex numbers and the addition, subtraction and absolute value of complex numbers in the complex plane
Mathematics	9-12	Algebra and Functions	138	Determine how the graph of a parabola changes if a , b and c changes in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola when given sufficient information

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Algebra and Functions	139	State, prove, and use De Moivre's Theorem

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Geometry	1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc
Mathematics	9-12	Geometry	2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch)
Mathematics	9-12	Geometry	3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself
Mathematics	9-12	Geometry	4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments
Mathematics	9-12	Geometry	5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another
Mathematics	9-12	Geometry	6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent
Mathematics	9-12	Geometry	7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent
Mathematics	9-12	Geometry	8	Explain how the criteria for triangle congruence (ASA, SAS, ASAand SSS) follow from the definition of congruence in terms of rigid motions
Mathematics	9-12	Geometry	9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints
Mathematics	9-12	Geometry	10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point
Mathematics	9-12	Geometry	11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals
Mathematics	9-12	Geometry	12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line
Mathematics	9-12	Geometry	13	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. Verify experimentally the properties of dilations given by a center and a scale factor: The dilation of a line segment is longer or shorter in the ratio given by the scale factor
Mathematics	9-12	Geometry	14	Given two figures, use the definition of similarity in terms of similarity transformations such as the AA criterion to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides
Mathematics	9-12	Geometry	15	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity
Mathematics	9-12	Geometry	16	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Geometry	17	Explain and use the relationship between the sine and cosine of complementary angles
Mathematics	9-12	Geometry	18	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems
Mathematics	9-12	Geometry	19	Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side
Mathematics	9-12	Geometry	20	Prove the Laws of Sines and Cosines and use them to solve problem
Mathematics	9-12	Geometry	21	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces)
Mathematics	9-12	Geometry	22	Prove that all circles are similar
Mathematics	9-12	Geometry	23	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle
Mathematics	9-12	Geometry	24	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle
Mathematics	9-12	Geometry	25	Construct a tangent line from a point outside a given circle to the circle
Mathematics	9-12	Geometry	26	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector
Mathematics	9-12	Geometry	27	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation
Mathematics	9-12	Geometry	28	Derive the equation of a parabola given a focus and directrix
Mathematics	9-12	Geometry	29	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant
Mathematics	9-12	Geometry	30	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point)
Mathematics	9-12	Geometry	31	Find the point on a directed line segment between two given points that partitions the segment in a given ratio
Mathematics	9-12	Geometry	32	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula
Mathematics	9-12	Geometry	33	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments
Mathematics	9-12	Geometry	34	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures
Mathematics	9-12	Geometry	35	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems
Mathematics	9-12	Geometry	36	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects
Mathematics	9-12	Geometry	37	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)
Mathematics	9-12	Geometry	38	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios)
Mathematics	9-12	Geometry	39	Understand the differences among supporting evidence, counterexamples and actual proofs
Mathematics	9-12	Geometry	40	Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems
Mathematics	9-12	Geometry	41	Identify and describe convex, concave, and regular polygons
Mathematics	9-12	Geometry	42	Use properties of congruent and similar polygons to solve problems
Mathematics	9-12	Geometry	43	Find and use measures of sides, perimeters, and areas of polygons. Relate these measures to each other using formulas
Mathematics	9-12	Geometry	44	Use coordinate geometry to prove properties of polygons such as regularity, congruence, and similarity

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Geometry	45	Describe, classify, and understand relationships among the quadrilaterals square, rectangle, rhombus, parallelogram, trapezoid, and kite
Mathematics	9-12	Geometry	46	Use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas.
Mathematics	9-12	Geometry	47	Find and use measures of sides, perimeters, and areas of quadrilaterals. Relate these measures to each other using formulas
Mathematics	9-12	Geometry	48	Identify and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular
Mathematics	9-12	Geometry	49	Define, identify, and construct altitudes, medians, angle bisectors, and perpendicular bisectors
Mathematics	9-12	Geometry	50	Use properties of congruent and similar triangles to solve problems involving lengths and areas
Mathematics	9-12	Geometry	51	Prove and apply theorems involving segments divided proportionally
Mathematics	9-12	Geometry	52	Prove that triangles are congruent or similar and use the concept of corresponding parts of congruent triangles
Mathematics	9-12	Geometry	53	Find and use measures of sides, perimeters, and areas of triangles. Relate these measures to each other using formulas
Mathematics	9-12	Geometry	54	Prove, understand, and apply the inequality theorems: triangle inequality, inequality in one triangle, and the hinge theorem
Mathematics	9-12	Geometry	55	State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle
Mathematics	9-12	Geometry	56	Use special right triangles (30° - 60° and 45° - 45°) to solve problems
Mathematics	9-12	Geometry	57	Define and use the trigonometric functions (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles
Mathematics	9-12	Geometry	58	Know and use the relationship $\sin^2 x + \cos^2 x = 1$
Mathematics	9-12	Geometry	59	Solve word problems involving right triangles
Mathematics	9-12	Geometry	60	Define, identify and use relationships among: radius, diameter, arc, measure of an arc, chord, secant, and tangent
Mathematics	9-12	Geometry	61	Define, find, and use measures of arcs and related angles (central, inscribed, and intersections of secants and tangents)
Mathematics	9-12	Geometry	62	Define and identify congruent and concentric circles
Mathematics	9-12	Geometry	63	Define, find, and use measures of circumference, arc length, and areas of circles and sectors. Use these measures to solve problems
Mathematics	9-12	Geometry	64	Describe the polyhedron that can be made from a given net (or pattern). Describe the net for a given polyhedron
Mathematics	9-12	Geometry	65	Describe relationships between the faces, edges, and vertices of polyhedra
Mathematics	9-12	Geometry	66	Describe symmetries of geometric solids
Mathematics	9-12	Geometry	67	Describe sets of points on spheres: chords, tangents, and great circles
Mathematics	9-12	Geometry	68	Identify and know properties of congruent and similar solids
Mathematics	9-12	Geometry	69	Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas
Mathematics	9-12	Geometry	70	State, use, and examine the validity of the converse, inverse, and contrapositive of “if – then” statements
Mathematics	9-12	Geometry	71	Identify and give examples of undefined terms, axioms, and theorems, and inductive and deductive proofs
Mathematics	9-12	Geometry	72	Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines by using appropriate geometric construction tools. Explain and justify the process used
Mathematics	9-12	Geometry	73	Recognize, use and justify the relationships between special pairs of angles formed by parallel lines and transversals
Mathematics	9-12	Geometry	74	Identify and apply properties of and theorems about parallel and perpendicular lines, write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines
Mathematics	9-12	Geometry	75	Identify, justify and apply properties of planes
Mathematics	9-12	Geometry	76	Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane
Mathematics	9-12	Geometry	77	Describe the intersection of two or more geometric figures in the plane
Mathematics	9-12	Geometry	78	Justifying the method used, find and use the sum of the measures of interior and exterior angles of convex polygons

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Geometry	79	Identify types of symmetry (i.e., line, point, rotational, self-congruences) of polygons
Mathematics	9-12	Geometry	80	Solve problems involving congruent and similar polygons
Mathematics	9-12	Geometry	81	Predict and describe the results of translations, reflections and rotations on polygons. Describe a motion or series of motions that will show that two shapes are congruent
Mathematics	9-12	Geometry	82	Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle
Mathematics	9-12	Geometry	83	Recognize and use coordinate geometry to verify properties of polygons such as regularity, congruence and similarity
Mathematics	9-12	Geometry	84	Prove and apply theorems about parallelograms and trapezoids (including isosceles trapezoids) involving their angles, sides and diagonals. Prove that the given quadrilaterals are parallelograms, rhombuses, rectangles, squares or trapezoids (as appropriate)
Mathematics	9-12	Geometry	85	Construct triangles congruent to given triangles. Explain and justify the process used
Mathematics	9-12	Geometry	86	Prove and apply the isosceles triangle theorem and its converse
Mathematics	9-12	Geometry	87	Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane
Mathematics	9-12	Geometry	88	Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions and determine whether the solutions are reasonable. Use technology as appropriate
Mathematics	9-12	Geometry	89	Construct the circle that passes through three given points not on a line. Construct tangents to circles. Circumscribe and inscribe circles. Justify the process used
Mathematics	9-12	Geometry	90	Develop simple geometric proofs involving circles and provide reasons for each statement
Mathematics	9-12	Geometry	91	Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres
Mathematics	9-12	Geometry	92	Solve problems involving congruent and similar solids
Mathematics	9-12	Geometry	93	Visualize solids and surfaces in three-dimensional space when given two-dimensional representations, and create two-dimensional representations for the surfaces of three-dimensional objects
Mathematics	9-12	Geometry	94	Describe the structure of and relationships within an axiomatic system (undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems)
Mathematics	9-12	Geometry	95	Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true. Illustrate its counterparts in other geometries
Mathematics	9-12	Geometry	96	Develop simple geometric proofs (i.e., direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs and flow charts formats. Provide reasons for each statement in the proofs

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Measurement	1	Find measures of interior and exterior angles of polygons, justifying the method used
Mathematics	9-12	Measurement	2	Define, find, and use measures of circumference, arc length, and areas of circles and sectors
Mathematics	9-12	Measurement	3	Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Data Analysis, Stats, and Probability	1	Define arithmetic and geometric sequences and series
Mathematics	9-12	Data Analysis, Stats, and Probability	2	Find specified terms of arithmetic and geometric sequences
Mathematics	9-12	Data Analysis, Stats, and Probability	3	Find partial sums of arithmetic and geometric series
Mathematics	9-12	Data Analysis, Stats, and Probability	4	Solve word problems involving applications of sequences and series
Mathematics	9-12	Data Analysis, Stats, and Probability	5	Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions
Mathematics	9-12	Data Analysis, Stats, and Probability	6	Understand the concepts of sample space and the probability distribution and construct sample spaces and distributions in simple cases
Mathematics	9-12	Data Analysis, Stats, and Probability	7	Use simulations to construct empirical probability distributions
Mathematics	9-12	Data Analysis, Stats, and Probability	8	Compute and interpret the expected value of random variables in simple cases
Mathematics	9-12	Data Analysis, Stats, and Probability	9	Understand the concepts of conditional probability and independent events
Mathematics	9-12	Data Analysis, Stats, and Probability	10	Understand how to compute the probability of a compound event
Mathematics	9-12	Data Analysis, Stats, and Probability	11	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent
Mathematics	9-12	Data Analysis, Stats, and Probability	12	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B
Mathematics	9-12	Data Analysis, Stats, and Probability	13	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities
Mathematics	9-12	Data Analysis, Stats, and Probability	14	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions
Mathematics	9-12	Data Analysis, Stats, and Probability	15	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution
Mathematics	9-12	Data Analysis, Stats, and Probability	16	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value
Mathematics	9-12	Data Analysis, Stats, and Probability	17	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values
Mathematics	9-12	Data Analysis, Stats, and Probability	18	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game)
Mathematics	9-12	Data Analysis, Stats, and Probability	19	Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events
Mathematics	9-12	Data Analysis, Stats, and Probability	20	Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risks of events
Mathematics	9-12	Data Analysis, Stats, and Probability	21	Use counting techniques to solve probability problems

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Data Analysis, Stats, and Probability	22	Use simulations to solve counting and probability problems
Mathematics	9-12	Data Analysis, Stats, and Probability	23	Understand and use the addition rule to calculate probabilities for mutually exclusive and nonmutually exclusive events
Mathematics	9-12	Data Analysis, Stats, and Probability	24	Understand and use the multiplication rule to calculate probabilities for independent and dependent events
Mathematics	9-12	Data Analysis, Stats, and Probability	25	Calculate the probabilities of complementary events
Mathematics	9-12	Data Analysis, Stats, and Probability	26	Understand conditional probability and Bayes' Theorem and use them to solve problems
Mathematics	9-12	Data Analysis, Stats, and Probability	27	Select and use appropriate statistical methods to analyze data
Mathematics	9-12	Data Analysis, Stats, and Probability	28	Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions
Mathematics	9-12	Data Analysis, Stats, and Probability	29	Understand how sample statistics reflect the values of the population parameters and use sampling distributions as the basis for informal inference
Mathematics	9-12	Data Analysis, Stats, and Probability	30	Understand how basic statistical techniques are used to monitor process characteristics in the workplace
Mathematics	9-12	Data Analysis, Stats, and Probability	31	Organize and display univariate and bivariate data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (i.e., mean, median) and spread (i.e., range, percentiles, variance, standard deviation). Understand the effects outliers on the data.
Mathematics	9-12	Data Analysis, Stats, and Probability	32	Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
Mathematics	9-12	Data Analysis, Stats, and Probability	33	Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each
Mathematics	9-12	Data Analysis, Stats, and Probability	34	Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable
Mathematics	9-12	Data Analysis, Stats, and Probability	35	Display and discuss bivariate data where at least one variable is categorical
Mathematics	9-12	Data Analysis, Stats, and Probability	36	Recognize how linear transformations of univariate data affect shape, center, and spread
Mathematics	9-12	Data Analysis, Stats, and Probability	37	Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
Mathematics	9-12	Data Analysis, Stats, and Probability	38	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve
Mathematics	9-12	Data Analysis, Stats, and Probability	39	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data
Mathematics	9-12	Data Analysis, Stats, and Probability	40	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related
Mathematics	9-12	Data Analysis, Stats, and Probability	41	Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models
Mathematics	9-12	Data Analysis, Stats, and Probability	42	Informally assess the fit of a function by plotting and analyzing residuals

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Data Analysis, Stats, and Probability	43	Fit a linear function for a scatter plot that suggests a linear association
Mathematics	9-12	Data Analysis, Stats, and Probability	44	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data
Mathematics	9-12	Data Analysis, Stats, and Probability	45	Compute (using technology) and interpret the correlation coefficient of a linear fit
Mathematics	9-12	Data Analysis, Stats, and Probability	46	Distinguish between correlation and causation
Mathematics	9-12	Data Analysis, Stats, and Probability	47	Understand statistics as a process for making inferences about population parameters based on a random sample from that population
Mathematics	9-12	Data Analysis, Stats, and Probability	48	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation
Mathematics	9-12	Data Analysis, Stats, and Probability	49	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling
Mathematics	9-12	Data Analysis, Stats, and Probability	50	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”)
Mathematics	9-12	Data Analysis, Stats, and Probability	51	Compare data sets using graphs and summary statistics
Mathematics	9-12	Data Analysis, Stats, and Probability	52	Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results
Mathematics	9-12	Data Analysis, Stats, and Probability	53	Evaluate reports based on data by considering the source of the data, the design of the study, the way the data are analyzed and displayed and whether the report confuses correlation with causation
Mathematics	9-12	Data Analysis, Stats, and Probability	54	Evaluate information based on data by considering the source of the data, the design of the study, the way the data are analyzed and displayed, and whether the report confuses correlation with causation
Mathematics	9-12	Data Analysis, Stats, and Probability	55	Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples
Mathematics	9-12	Data Analysis, Stats, and Probability	56	Make predictions from the least squares regression line or its equation
Mathematics	9-12	Data Analysis, Stats, and Probability	57	Explore the geometric, or waiting-time, distribution
Mathematics	9-12	Data Analysis, Stats, and Probability	58	Understand and apply basic ideas related to the design and interpretation of surveys, such as background information, random sampling, and bias
Mathematics	9-12	Data Analysis, Stats, and Probability	59	Construct simulated sampling distributions of sample proportions and use sampling distributions to identify which proportions are likely to be found in a sample of a given size
Mathematics	9-12	Data Analysis, Stats, and Probability	60	Construct and interpret margin of error and confidence intervals for population proportions
Mathematics	9-12	Data Analysis, Stats, and Probability	61	Use networks, traceable paths, tree diagrams, Venn diagrams, and other pictorial representations to find the number of outcomes in a problem situation
Mathematics	9-12	Data Analysis, Stats, and Probability	62	Use the fundamental counting principle to find the number of outcomes in a problem situation
Mathematics	9-12	Data Analysis, Stats, and Probability	63	Use combinatorial reasoning to solve problems
Mathematics	9-12	Data Analysis, Stats, and Probability	64	Use election theory techniques to analyze election data

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Data Analysis, Stats, and Probability	65	Use weighted voting techniques to decide voting power within a group
Mathematics	9-12	Data Analysis, Stats, and Probability	66	Use fair division techniques to divide continuous objects
Mathematics	9-12	Data Analysis, Stats, and Probability	67	Use fair division techniques to solve apportionment problems
Mathematics	9-12	Data Analysis, Stats, and Probability	68	Use game theory to solve strictly determined games
Mathematics	9-12	Data Analysis, Stats, and Probability	69	Use game theory to solve nonstrictly determined games
Mathematics	9-12	Data Analysis, Stats, and Probability	70	Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge. Recognize an infinite series as the limit of a sequence of partial sums
Mathematics	9-12	Data Analysis, Stats, and Probability	71	Model and solve word problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable
Mathematics	9-12	Data Analysis, Stats, and Probability	72	Derive the binomial theorem by combinatorics
Mathematics	9-12	Data Analysis, Stats, and Probability	73	Find linear models by using median fit and least squares regression methods. Decide which among several linear models gives a better fit. Interpret the slope in terms of the original context
Mathematics	9-12	Data Analysis, Stats, and Probability	74	Find a quadratic, exponential, logarithmic, power, or sinusoidal function to model a data set and explain the parameters of the model
Mathematics	9-12	Data Analysis, Stats, and Probability	75	Define and use the mathematical induction method of proof
Mathematics	9-12	Data Analysis, Stats, and Probability	76	Create, compare, and evaluate different graphic displays of the same data, using histograms, frequency polygons, cumulative frequency distribution functions, pie charts, scatterplots, stem-and-leaf plots, and box-and-whisker plots. Draw these by hand or use a computer spreadsheet program
Mathematics	9-12	Data Analysis, Stats, and Probability	77	Compute and use mean, median, mode, weighted mean, geometric mean, harmonic mean, range, quartiles, variance, and standard deviation
Mathematics	9-12	Data Analysis, Stats, and Probability	78	Understand the counting principle, permutations, and combinations and use them to solve problems
Mathematics	9-12	Data Analysis, Stats, and Probability	79	Understand the central limit theorem and use it to solve problems
Mathematics	9-12	Data Analysis, Stats, and Probability	80	Compute and use confidence intervals to make estimates

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Discrete Mathematics	1	Understand matrices as systems that have some of the properties of the real-number system
Mathematics	9-12	Discrete Mathematics	2	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network
Mathematics	9-12	Discrete Mathematics	3	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled
Mathematics	9-12	Discrete Mathematics	4	Add, subtract, and multiply matrices of appropriate dimensions
Mathematics	9-12	Discrete Mathematics	5	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties
Mathematics	9-12	Discrete Mathematics	6	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse
Mathematics	9-12	Discrete Mathematics	7	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors
Mathematics	9-12	Discrete Mathematics	8	Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area
Mathematics	9-12	Discrete Mathematics	9	Use an adjacency matrix to describe a vertex-edge graph
Mathematics	9-12	Discrete Mathematics	10	Perform row and column sums for matrix equations
Mathematics	9-12	Discrete Mathematics	11	Use matrices to organize and display data in a variety of real-world settings
Mathematics	9-12	Discrete Mathematics	12	Develop mathematical modeling skills by building matrix models and then apply the models to solve problems
Mathematics	9-12	Discrete Mathematics	13	Use matrices and inverse matrices to answer questions that involve systems of linear equations
Mathematics	9-12	Discrete Mathematics	14	Build and use matrix representations to model polygons, transformations, and computer animations
Mathematics	9-12	Discrete Mathematics	15	Use the properties of matrix multiplication, including identity and inverse matrices, to solve problems
Mathematics	9-12	Discrete Mathematics	16	Understand vectors as systems that have some of the properties of the real-number system
Mathematics	9-12	Discrete Mathematics	17	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., \mathbf{v} , $ \mathbf{v} $, $\ \mathbf{v}\ $, v)
Mathematics	9-12	Discrete Mathematics	18	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point
Mathematics	9-12	Discrete Mathematics	19	Solve problems involving velocity and other quantities that can be represented by vectors
Mathematics	9-12	Discrete Mathematics	20	Add and subtract vectors
Mathematics	9-12	Discrete Mathematics	21	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes
Mathematics	9-12	Discrete Mathematics	22	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum
Mathematics	9-12	Discrete Mathematics	23	Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of \mathbf{w} , with the same magnitude as \mathbf{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise
Mathematics	9-12	Discrete Mathematics	24	Multiply a vector by a scalar
Mathematics	9-12	Discrete Mathematics	25	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\mathbf{v}_x, \mathbf{v}_y) = (c\mathbf{v}_x, c\mathbf{v}_y)$
Mathematics	9-12	Discrete Mathematics	26	Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\ c\mathbf{v}\ = c \mathbf{v} $. Compute the direction of $c\mathbf{v}$ knowing that when $ c \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$)
Mathematics	9-12	Discrete Mathematics	27	Construct vertex-edge graph models involving relationships among a finite number of elements
Mathematics	9-12	Discrete Mathematics	28	Construct digraphs
Mathematics	9-12	Discrete Mathematics	29	Use Euler paths and circuits to solve real-world problems
Mathematics	9-12	Discrete Mathematics	30	Develop the skill of algorithmic problem solving: designing, using, and analyzing systematic procedures for problem solving
Mathematics	9-12	Discrete Mathematics	31	Use a recursion function to describe an exponential function

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Discrete Mathematics	32	Use a variety of recursion equations to describe a function
Mathematics	9-12	Discrete Mathematics	33	Analyze and interpret relationships defined iteratively and recursively
Mathematics	9-12	Discrete Mathematics	34	Define arithmetic and geometric sequences recursively
Mathematics	9-12	Discrete Mathematics	35	Use a recursion function to describe a fractal
Mathematics	9-12	Discrete Mathematics	36	Use iteration and recursion as tools to represent, analyze, and solve problems involving sequential change
Mathematics	9-12	Discrete Mathematics	37	Understand and apply recursion equations, particularly combined recursion equations of the form $A_n = rA_{n-1} + b$.
Mathematics	9-12	Discrete Mathematics	38	Model and solve problems involving patterns using recursion and iteration, growth and decay, and compound interest
Mathematics	9-12	Discrete Mathematics	39	Use recursive thinking to solve problems
Mathematics	9-12	Discrete Mathematics	40	Analyze and apply algorithms for searching (sequential, binary), for sorting (bubble sort, quick sort, bin sort) and for solving optimization problems
Mathematics	9-12	Discrete Mathematics	41	Determine the number of ways events can occur using permutations, combinations and the Fundamental Counting Principle
Mathematics	9-12	Discrete Mathematics	42	Determine whether two propositions are logically equivalent
Mathematics	9-12	Discrete Mathematics	43	Distinguish between inductive and deductive reasoning. Identify inductive reasoning as central to the scientific method and deductive reasoning as characteristic of mathematics
Mathematics	9-12	Discrete Mathematics	44	Experience in mathematical modeling by building and using vertex-edge graph models to solve problems in a variety of real-world settings
Mathematics	9-12	Discrete Mathematics	45	Optimize networks in different ways and in different contexts by finding minimal spanning trees, shortest paths, and Hamiltonian paths
Mathematics	9-12	Discrete Mathematics	46	Use and interpret relational conjunctions (and, or, not), terms of causation (if... then) and equivalence (if and only if). Distinguish between the common uses of such terms in everyday language and their use in mathematics
Mathematics	9-12	Discrete Mathematics	47	Use truth tables to determine the truth values of propositional statements
Mathematics	9-12	Discrete Mathematics	48	Recognize syllogisms, tautologies, flawed reasoning and circular reasoning
Mathematics	9-12	Discrete Mathematics	49	Construct and interpret directed and undirected graphs, decision trees, networks and flow charts
Mathematics	9-12	Discrete Mathematics	50	Use critical-path analysis to solve scheduling problems
Mathematics	9-12	Discrete Mathematics	51	Explore function iteration and, in the process, informally introduce function composition
Mathematics	9-12	Discrete Mathematics	52	Know and use the concepts of sets, elements and subsets
Mathematics	9-12	Discrete Mathematics	53	Perform operations on sets (union, intersection, complement, cross product)
Mathematics	9-12	Discrete Mathematics	54	Identify and give examples of undefined terms, axioms and theorems
Mathematics	9-12	Discrete Mathematics	55	Describe logical statements using the terms assumption, hypothesis, conclusion, converse, inverse and contrapositive. Find the converse, inverse and contrapositive of statements
Mathematics	9-12	Discrete Mathematics	56	Explain and illustrate the role of definitions, conjectures, theorems, proofs and counterexamples in mathematical reasoning. Construct logical arguments, assess the validity of logical arguments and give counterexamples to disprove statements
Mathematics	9-12	Discrete Mathematics	57	Use mathematical induction to prove simple propositions
Mathematics	9-12	Discrete Mathematics	58	Use graph-coloring techniques to solve problems
Mathematics	9-12	Discrete Mathematics	59	Use bin-packing techniques to solve problems of optimizing resource usage
Mathematics	9-12	Discrete Mathematics	60	Convert between a pair of parametric equations and an equation in x and y
Mathematics	9-12	Discrete Mathematics	61	Analyze planar curves, including those given in parametric form
Mathematics	9-12	Discrete Mathematics	62	Model and solve problems using parametric equations
Mathematics	9-12	Discrete Mathematics	63	Use row-reduction techniques to solve problems
Mathematics	9-12	Discrete Mathematics	64	Use Markov chains to solve problems
Mathematics	9-12	Discrete Mathematics	65	Use finite differences to solve problems
Mathematics	9-12	Discrete Mathematics	66	Use graphs consisting of vertices and edges to model a problem situation
Mathematics	9-12	Discrete Mathematics	67	Use critical path analysis to solve scheduling problems
Mathematics	9-12	Discrete Mathematics	68	Use minimal spanning trees to solve problems

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Discrete Mathematics	69	Use geometric techniques to solve optimization problems
Mathematics	9-12	Discrete Mathematics	70	Use the Simplex method to solve optimization problems with and without technology

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Calculus	1	Understand the concept of limit and estimate limits from graphs and tables of values
Mathematics	9-12	Calculus	2	Find limits by substitution
Mathematics	9-12	Calculus	3	Find limits of sums, differences, products, and quotients
Mathematics	9-12	Calculus	4	Find limits of rational functions that are undefined at a point
Mathematics	9-12	Calculus	5	Find one-sided limits
Mathematics	9-12	Calculus	6	Find limits at infinity
Mathematics	9-12	Calculus	7	Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior
Mathematics	9-12	Calculus	8	Find special limits such as $\lim_{x \rightarrow 0} \frac{\sin x}{x}$. Example: Use a diagram to show that the limit above is equal to 1.
Mathematics	9-12	Calculus	9	Understand continuity in terms of limits
Mathematics	9-12	Calculus	10	Decide if a function is continuous at a point
Mathematics	9-12	Calculus	11	Find the types of discontinuities of a function
Mathematics	9-12	Calculus	12	Understand and use the Intermediate Value Theorem on a function over a closed interval
Mathematics	9-12	Calculus	13	Understand and apply the Extreme Value Theorem: If $f(x)$ is continuous over a closed interval, then f has a maximum and a minimum on the interval
Mathematics	9-12	Calculus	14	Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as a rate of change
Mathematics	9-12	Calculus	15	State, understand, and apply the definition of derivative
Mathematics	9-12	Calculus	16	Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions
Mathematics	9-12	Calculus	17	Find the derivatives of sums, products, and quotients
Mathematics	9-12	Calculus	18	Find the derivatives of composite functions, using the chain rule
Mathematics	9-12	Calculus	19	Find the derivatives of implicitly-defined functions
Mathematics	9-12	Calculus	20	Find derivatives as inverse functions
Mathematics	9-12	Calculus	21	Find second derivatives and derivatives of higher order
Mathematics	9-12	Calculus	22	Find derivatives using logarithmic differentiation
Mathematics	9-12	Calculus	23	Understand and use the relationship between differentiability and continuity
Mathematics	9-12	Calculus	24	Understand and apply the Mean Value Theorem
Mathematics	9-12	Calculus	25	Find the slope of a curve at a point, including points at which there are vertical tangents and no tangents
Mathematics	9-12	Calculus	26	Find a tangent line to a curve at a point and a local linear approximation
Mathematics	9-12	Calculus	27	Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of f and the sign of f'
Mathematics	9-12	Calculus	28	Find local and absolute maximum and minimum points
Mathematics	9-12	Calculus	29	Analyze curves, including the notions of monotonicity and concavity
Mathematics	9-12	Calculus	30	Find points of inflection of functions. Understand the relationship between the concavity of f and the sign of f'' . Understand points of inflection as places where concavity changes
Mathematics	9-12	Calculus	31	Use first and second derivatives to help sketch graphs. Compare the corresponding characteristics of the graphs of f , f' , and f''
Mathematics	9-12	Calculus	32	Use implicit differentiation to find the derivative of an inverse function
Mathematics	9-12	Calculus	33	Solve optimization problems

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Calculus	34	Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed, and acceleration
Mathematics	9-12	Calculus	35	Find the velocity and acceleration of a particle moving in a straight line
Mathematics	9-12	Calculus	36	Model rates of change, including related rates problems
Mathematics	9-12	Calculus	37	Use rectangle approximations to find approximate values of integrals.
Mathematics	9-12	Calculus	38	Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points
Mathematics	9-12	Calculus	39	Interpret a definite integral as a limit of Riemann Sums.
Mathematics	9-12	Calculus	40	Understand the Fundamental Theorem of Calculus: Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is $f'(x)dx = f(b) - f(a)$.
Mathematics	9-12	Calculus	41	Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined
Mathematics	9-12	Calculus	42	<p>Understand and use these properties of definite integrals:</p> $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ $\int_a^b k \cdot f(x)dx = k \int_a^b f(x)dx$ $\int_a^a f(x)dx = 0$ $\int_a^b f(x)dx = -\int_b^a f(x)dx$ $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ <p>If $f(x) \leq g(x)$ on $[a, b]$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$.</p> <p>Example: Find $\int_0^3 5x^2 dx$, given that $\int_0^3 x^2 dx = 9$.</p>
Mathematics	9-12	Calculus	43	Understand and use integration by substitution (or change of variable) to find values of integrals
Mathematics	9-12	Calculus	44	Understand and use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values
Mathematics	9-12	Calculus	45	Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line
Mathematics	9-12	Calculus	46	Solve separable differential equations and use them in modeling
Mathematics	9-12	Calculus	47	Solve differential equations of the form $y' = ky$ as applied to growth and decay problems.
Mathematics	9-12	Calculus	48	Use definite integrals to find the area between a curve and the x-axis, or between two curves
Mathematics	9-12	Calculus	49	Use definite integrals to find the average value of a function over a closed interval
Mathematics	9-12	Calculus	50	Use definite integrals to find the volume of a solid with known cross-sectional area
Mathematics	9-12	Calculus	51	Apply integration to model and solve problems in physics, biology, economics, etc., using the integral as a rate of change to give accumulated change and using the method of setting up an approximating Riemann Sum and representing its limit as a definite integral

MATHEMATICS: HIGH SCHOOL

Content Area	Grade Level/Span	Strand	Number	Content Area Topic
Mathematics	9-12	Process, Prob. Solving, Practice	1	Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards
Mathematics	9-12	Process, Prob. Solving, Practice	2	Decide whether a solution is reasonable in the context of the original situation
Mathematics	9-12	Process, Prob. Solving, Practice	3	Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution set as the original. Understand that similar logic applies to solving systems of equations simultaneously
Mathematics	9-12	Process, Prob. Solving, Practice	4	Decide whether a given algebraic statement is true always, sometimes, or never (statements involving linear or quadratic expressions, equations, or inequalities)
Mathematics	9-12	Process, Prob. Solving, Practice	5	Distinguish between inductive and deductive reasoning, identifying and providing examples of each
Mathematics	9-12	Process, Prob. Solving, Practice	6	Identify and give examples of undefined terms, axioms, and theorems
Mathematics	9-12	Process, Prob. Solving, Practice	7	Construct logical arguments, judge their validity, and give counterexamples to disprove statements
Mathematics	9-12	Process, Prob. Solving, Practice	8	Monitor and reflect on the process of mathematical problem solving
Mathematics	9-12	Process, Prob. Solving, Practice	9	Recognize, develop, evaluate, select and use various types of mathematical arguments, reasoning and methods of proofs as fundamental aspects of mathematics, make and investigate mathematical conjectures
Mathematics	9-12	Process, Prob. Solving, Practice	10	Communicate both verbally and in writing your mathematical thinking coherently and clearly to peers, teachers and others
Mathematics	9-12	Process, Prob. Solving, Practice	11	Analyze and evaluate the mathematical thinking and strategies of others
Mathematics	9-12	Process, Prob. Solving, Practice	12	Use the language of mathematics to express mathematical ideas properly
Mathematics	9-12	Process, Prob. Solving, Practice	13	Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
Mathematics	9-12	Process, Prob. Solving, Practice	14	Recognize and apply mathematics in contexts outside of mathematics
Mathematics	9-12	Process, Prob. Solving, Practice	15	Determine appropriate accuracy and precision of measurements in problem situations
Mathematics	9-12	Process, Prob. Solving, Practice	16	The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning and problem-solving skills - Elementary students should learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator- The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself
Mathematics	9-12	Process, Prob. Solving, Practice	17	Make sense of problems and persevere in solving them
Mathematics	9-12	Process, Prob. Solving, Practice	18	Reason abstractly and quantitatively
Mathematics	9-12	Process, Prob. Solving, Practice	19	Construct viable arguments and critique the reasoning of others
Mathematics	9-12	Process, Prob. Solving, Practice	20	Model with mathematics
Mathematics	9-12	Process, Prob. Solving, Practice	21	Use appropriate tools strategically
Mathematics	9-12	Process, Prob. Solving, Practice	22	Attend to precision
Mathematics	9-12	Process, Prob. Solving, Practice	23	Look for and make use of structure
Mathematics	9-12	Process, Prob. Solving, Practice	24	Look for and express regularity in repeated reasoning